\( \sum F = \overrightarrow{0} \quad -T_2 + T_B + T_R = 0 \) \hspace{1cm} (1)

Note that although we have assumed \( T_R \) as shown, it really is opposite. More about this later.

\[
\begin{align*}
T_1 &= T_L \\
T_2 &= T_R \\
T_3 &= T_R
\end{align*}
\]

\( T_2 = T_3 \)

\( \varphi_1 + \varphi_2 + \varphi_3 = 0 \)

\[
\frac{T_1 L_1}{I_p G_1} + \frac{T_2 L_2}{I_p G_2} + \frac{T_3 L_3}{I_p G_3} = 0 \quad (2)
\]
Section (1)

\[ T_{all} = 50 \ \text{ksi} \]
\[ d_0 = 3.5 \]
\[ d_c = 3.26 \]
\[ G = 12.5 \times 10^6 \ \text{psi} \]

Section (2)

\[ T_{rec} = 50 \ \text{ksi} \]
\[ d_0 = 3.5 \]
\[ d_c = 3.26 \]
\[ G = 12.5 \times 10^6 \ \text{psi} \]

Section (3)

\[ T_{rec} = 18 \ \text{ksi} \]
\[ d_0 = 2 \]
\[ G = 5.6 \times 10^6 \ \text{psi} \]
Rewrite eqn. # 0

\[-T_1 + T_B + T_3 = 0\]

Rewrite eqn. # 2 or \[T_B = T_1 - T_3\]

\[\frac{T_1 L_1}{I_p \sigma} + T_3 \left( \frac{L_2}{I_p 2 \sigma} + \frac{T_3}{I_p \sigma} \right) = 0\]

It appears that we have 3 unknowns. However, we know the stress & Torque eqns.

\[T = \frac{T \sqrt{d/2}}{I_p}\]

The key to this problem is to understand that while one of the sections may be at the allowable stress, the other two will be less than the allowable stress, if the structure is safe.
We will have to guess which one, and check our answers.

Assume section (3) is critical:

\[ |T_3| = \frac{T_3 I_{P_3}}{d/2} = \frac{18 \times 10^3 \pi}{4/2} \]

\[ |T_3| = 28.27 \text{ kip-in} \]

If so, \( T_2 = T_3 = 28.27 \text{ kip-in} \)

\( T_2 = T_1 - T_3 \)

Note that \( T_1 \) & \( T_3 \) must have opposite signs.

\[ T_1 + T_3 \left( \frac{L_2}{I_{P_2} G_2} + \frac{L_3}{I_{P_3} G_3} \right) \frac{I_{P_1} G_1}{L_1} = 0 \]
\[ T_1 + T_3 \left( \frac{L_2}{L_1} \frac{E_{p1}}{E_2} \frac{G_1}{G_2} + \frac{L_3}{L_1} \frac{E_{p1}}{E_3} \frac{G_1}{G_3} \right) = 0 \]

\[ T_1 + T_3 \left( \frac{22}{30} + \frac{18}{30} \frac{\pi 132 \left[ 3.5^4 - 3.26^4 \right]}{\pi 132^{2/3} \frac{12.5}{5.6}} \right) = 0 \]

\[ T_1 + 3.84 T_3 = 0 \]

\[ T_1 = -3.84 T_3 \implies |T_1| = 108.6 \text{ kip} \]

Check stress in 1 & 2:

\[ T_1 = \frac{(108.6 \times 10^3) \frac{3.5}{2}}{\frac{\pi}{32} (3.5^4 - 3.26^4)} = 52.14 \text{ ksi} \geq 750 \text{ ksi} \]

\[ \text{Not Safe!} \]

\[ T_2 = \frac{(28.27 \times 10^3) \frac{3.5}{2}}{\frac{\pi}{32} (3.5^4 - 3.26^4)} = 13.56 \text{ ksi} \]
Suppose element (2) controls:

\[ T_2 = \frac{T_2}{I_{p2}} \]

\[ T_2 = \frac{(50 \text{ kips}) \pi^{2} (3.5^4 - 3.26^4)}{3.5^2 \pi^2} \]

\[ T_2 = 104.1 \text{ kips in} \]

This is even higher than when section (3) controlled the structure, "not safe!"

So, section (1) must control.

\[ T_1 = \frac{(60 \text{ kips}) \pi^{2} (3.5^4 - 3.66^4)}{3.5^2 \pi^2} \]

\[ T_1 = 104.1 \text{ kips in} \]
If $T_1 = 104.1$ kip \cdot in

* $T_1 = -3.84T_3 \quad \text{then}$

$$T_3 = -27.11 \text{ kip} \cdot \text{in} = T_2$$

$$T_B = T_1 - T_3 = 104.1 - (-27.11)$$

$$T_B = 131.2 \text{ kip} \cdot \text{in}$$

\underline{Final Answer gives}

$T_1 = 50.0 \text{kfsi}$

$T_2 = 13.0 \text{kfsi}$

$T_3 = 17.26 \text{kfsi}$